$$

L\_n (x) = \sum\_{k=0}^n y\_k \frac{\omega\_{n+1}(x)}{(x-x\_k)\omega'\_{n+1}(x\_k)}

$$

其中

$$

\begin{align}

\omega\_{n+1}(x) &=(x-x\_0)(x-x\_1)\cdots(x-x\_n)\\

\omega'\_{n+1}(x\_k) &=(x\_k-x\_0)\cdots(x\_k-x\_{k-1})(x\_k-x\_{k+1})\cdots(x\_k-x\_n)

\end{align}

$$

当 $n=1$ 时，线性插值公式为:

$$

L\_1(x)=y\_k\frac{x\_{k+1}-x}{x\_{k+1}-x\_k}+y\_{k+1}\frac{x-x\_k}{x\_{k+1}-x\_k}

$$

当 $n=2$ 时，线性插值公式为:

$$

L\_2(x)=y\_{k-1}\frac{(x-x\_k)(x-x\_{k+1})}{(x\_{k-1}-x\_k)(x\_{k-1}-x\_{k+1})}+y\_k\frac{(x-x\_{k-1})(x-x\_{k+1})}{(x\_{k}-x\_{k-1})(x\_{k}-x\_{k+1})}+y\_{k+1}\frac{(x-x\_k)(x-x\_{k\_1})}{(x\_{k+1}-x\_{k-1})(x\_{k+1}-x\_{k})}

$$

\*\*例1\*\*

已知 $(u\_n,f\_n),(u\_{n+1},f\_{n+1})$,在四阶二步法:

$$

u\_{n+2}=u\_n + \frac{h}{3}(f\_{n+2}+4f\_{n+1}+f\_n)

$$

中用已知点代替未知点$(u\_{n+2},f\_{n+2})$

\*\*解 :\*\* 用 lagrange插值方法,$f\_{n+2}$可以表示为：

$$

\begin{align}

f\_{n+2} &=f\_n\frac{u\_{n+1}-u\_{n+2}}{u\_{n+1}-u\_n}+f\_{n+1}\frac{u\_{n+2}-u\_n}{u\_{n+1}-u\_n}\\

&= f\_n\frac{h}{-h}+f\_{n+1}\frac{2h}{h}\\

&=-f\_n+2f\_{n+1}

\end{align}

$$

其中

$$

u\_{n+2}=u\_{n+1}+hu'\_{n+1}+r(h)\\

u\_n=u\_{n+1}-hu'\_{n+1}+r(-h)\\

\frac{u\_{n+1}-u\_{n+2}}{u\_{n+1}-u\_n}=\frac{-hu'\_{n+1}-r(h)}{hu'\_{n+1}+r(h)}=-1\\

$$

同理可得

$$

\frac{u\_{n+2}-u\_n}{u\_{n+1}-u\_n}=\frac{2hu'\_n+r(2h)}{hu'\_n+r(h)}=2

$$

则

$$

\begin{align}

u\_{n+2} &=u\_n + \frac{h}{3}(f\_{n+2}+4f\_{n+1}+f\_n)\\

&=u\_n + \frac{h}{3}(-f\_n+2f\_{n+1}+4f\_{n+1}+f\_n)\\

&=u\_n+\frac{h}{3}(-f\_n+5f\_{n+1})

\end{align}

$$

\* 已知有很多点时,求未知点时,也可以用 Lagrange插值公式来求解.比如,已知(u\_{n-i},f\_{n-i})i=0,1,\cdots,k,在线性多步法中

$$

u\_{n+1}=u\_n+\sum\_{j=0}^k \beta\_j f\_{n+1-j}

$$

对于 $(u\_{n+1},f\_{n+1})$ 可以用已知点$(u\_{n-i},f\_{n-i})i=0,1,\cdots,k$,来代替求解未知.